éléments finis et ETR

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$$\left\| \begin{array}{l} \textbf{search } I(\boldsymbol{x}, \boldsymbol{s}) : \mathcal{D} \times \mathcal{S}^2 \mapsto \mathbb{R}^+ \\ \\ \boldsymbol{s} \cdot \nabla I(\boldsymbol{x}, \boldsymbol{s}) + (\kappa + \sigma_s) I(\boldsymbol{x}, \boldsymbol{s}) = \sigma_s \oint_{4\pi} \Phi(\boldsymbol{s}, \boldsymbol{s}') I(\boldsymbol{x}, \boldsymbol{s}') \, \mathrm{d}\boldsymbol{s}' + \kappa I_b(T) \\ \\ + \text{B.C.} \end{array} \right.$$

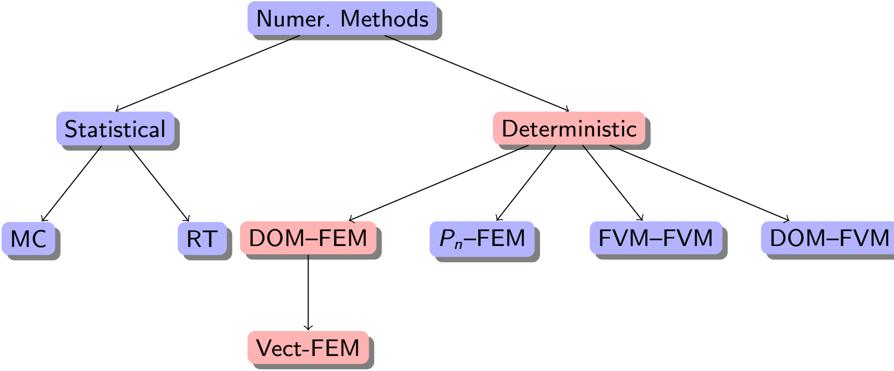
features :

- advection (transport)
- reaction
- no "diffusion"
- integro-difff eq.

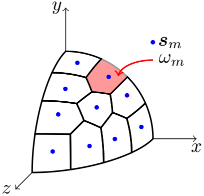
regularity

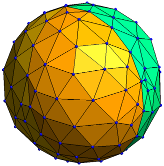
$$I(\boldsymbol{x}, \boldsymbol{s}) \in H^1(\mathcal{D}) \times L^2(\mathcal{S}^2)$$

classification



quadrature (ou discrétisation espace angulaire)





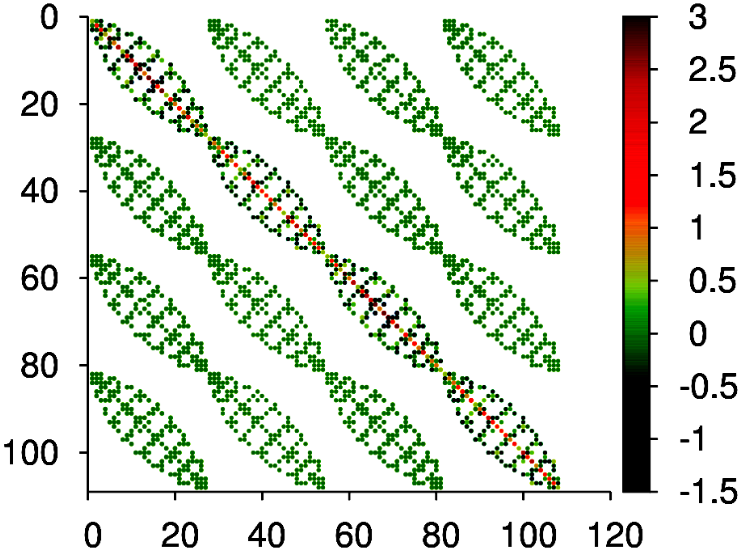
approximation du terme intégrale par une somme pondérée

$$\int_{\mathcal{S}} \Phi(\boldsymbol{s}, \boldsymbol{s}') I(\boldsymbol{x}, \boldsymbol{s}') \, \mathrm{d}\boldsymbol{s}' \simeq \sum_{j=1}^{N_d} \omega_j \Phi(\boldsymbol{s}, \boldsymbol{s}_j) I(\boldsymbol{x}, \boldsymbol{s}_j)$$

système à N_d équations à N_d inconnues, continues en espace

$$(E_m) : \boldsymbol{s}_m \cdot \nabla I_m(\boldsymbol{x}) + \beta I_m(\boldsymbol{x}) = \sigma_s \sum_{j=0}^{N_d} \omega_j \Phi_{m,j} I_j(\boldsymbol{x}) + \kappa I_b(T) \quad \forall m = 1, \cdots N_d$$

dom



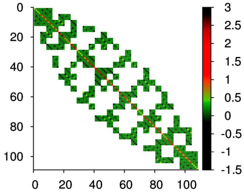
vectorial stabilized finite elements

- RTE set in vectorial form :

$$\mathbb{S} \cdot \nabla \mathbb{I} + \beta \mathbb{I} - \Theta \mathbb{I} = \kappa I_b \mathbb{1}$$

- weak form based on tensor product :

$$\int_{\mathcal{D}} [\mathbb{S} \cdot \nabla \mathbb{I} + \beta \mathbb{I} - \Theta \mathbb{I} - \kappa I_b \mathbb{1}]^\top (\mathbb{V} + \gamma \mathbb{S} \cdot \nabla \mathbb{I}) = 0$$



- Output is a single equation

- Weakens the coupling : short banded matrix

- One step building + one step solving

M.A. Badri, P. Jolivet, B. Rousseau, S. Le Corre, H. Digonnet et Y. Favennec. "Vectorial finite elements for solving the radiative transfer equation". In : Journal of Quantitative Spectroscopy and Radiative Transfer 212 (2018), p. 59–74

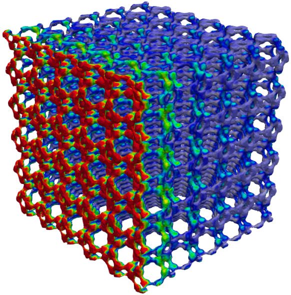
parallelization

What

- Means to **faster computation**
- Means to **larger computation**

Why

- Large scale problems
- Huge meshes
  - Complex geometry
  - Sharp solution fields
- Vectorial FEM for RTE
  - Inevitable in 3D



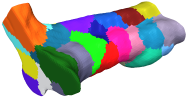
N_h	Overall solving time	
	1200 PU	1 PU
0.6 Billion	$\approx$ 14 (min)	$\approx$ 12 (days)

How

- Use multiple processing units (CPUs / cores / processes / threads,..)
  - **Assemble linear system in parallel** $\mathbb{A}\mathbb{I} = \mathbb{b}$ (crucial for RTE)
  - **Solve linear system in parallel** $\mathbb{I} = \underline{\mathbf{A}}^{-1}\mathbb{b}$

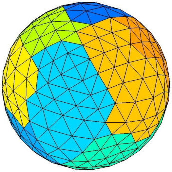
Two ways to parallelize RTE solving process
Since $I = I(\boldsymbol{x}, \mathcal{S})$

Domain Decomposition (DD)



- Work with $\{\boldsymbol{x} \in \Omega^h\}_{i=1}^{N_p}$

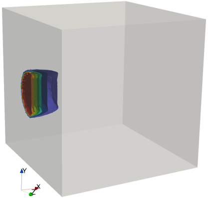
Angular Decomposition (AD)



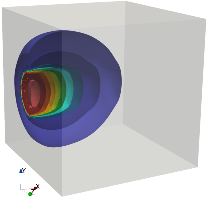
- Work with $\{\mathcal{S}_m \in \mathcal{S}^{N_d}\}_{i=1}^{N_p}$

- Both these methods are compatible with the vectorial FEM scheme

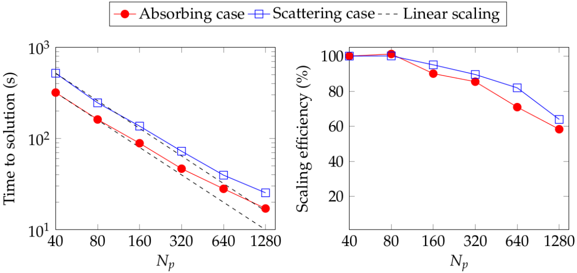
property invariant scaling for DD



Absorbing medium



Scattering medium



Test case 3D

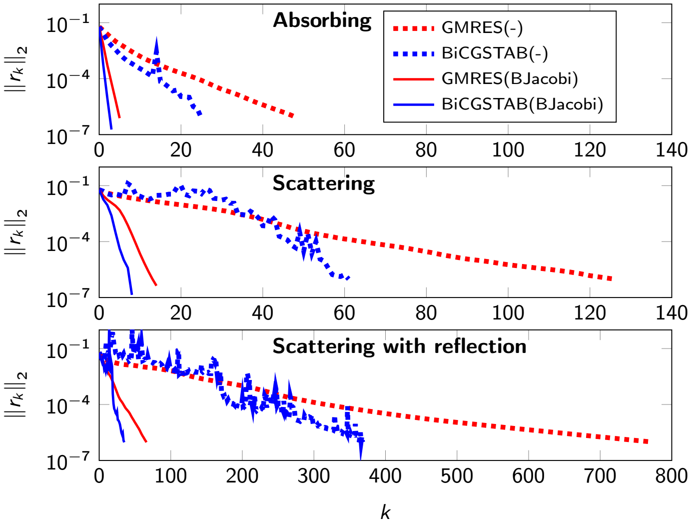
- $N_d = 80$
- $N_h = 0.3$ million
- 24 million unknowns
- GMRES with Jacobi

Advantage

- Quasi-linear scaling
- Property invariant scaling

M.A. Badri, P. Jolivet, B. Rousseau et Y. Favennec. "High Performance Computation of Radiative Transfer Equation Using the Finite Element Method". In : Journal of Computational Physic 360 (2018), p. 74–92

solver convergence

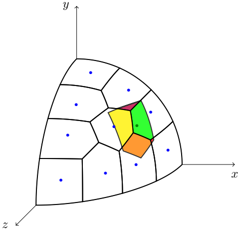


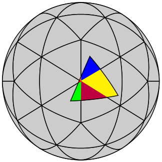
M.A. Badri, P. Jolivet, B. Rousseau et Y. Favennec. "Preconditioned Krylov sub-space methods for solving radiative transfer problems with scattering and reflection". In : Computers & Mathematics with Applications (2019)

$$I_m^{\vee}(\boldsymbol{x}) = \rho(\boldsymbol{s}_m \cdot \boldsymbol{n}) \sum_{i=1}^{N_d} \delta_{m,j}(\boldsymbol{n}) I_j(\boldsymbol{x})$$

$$\Longrightarrow \sum_{j \neq m} \left[\cdots + \int_{\partial \mathcal{D}^{m-}} \delta_{m,j}(\boldsymbol{n}) I_j v(\boldsymbol{s}_m \cdot \boldsymbol{n}) \, \mathrm{d}\Gamma \right]$$

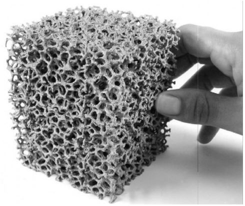
Méthode de partitionnement : $\delta_{m,j}(\boldsymbol{n}) = \dfrac{\text{mes}\ (\Omega_m \cap \Omega_j)}{\text{mes}(\Omega_m)}$

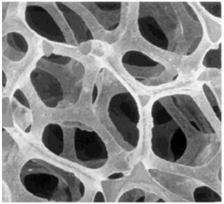


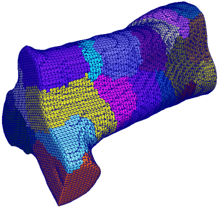


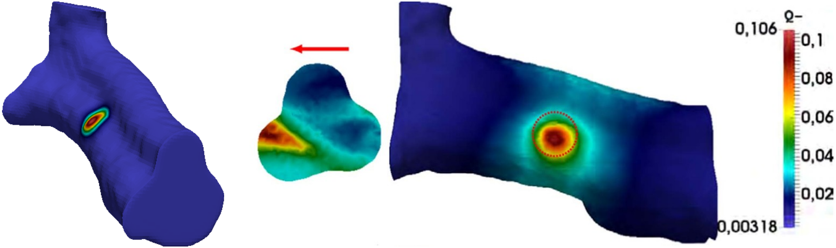
D. Le Hardy, Y. Favennec, B. Rousseau et F. Hecht. “Specular reflection treatment for the 3D radiative transfer equation solved with the discrete ordinates method”. In : Journal of Computational Physic 334 (2017), p. 541–572

application







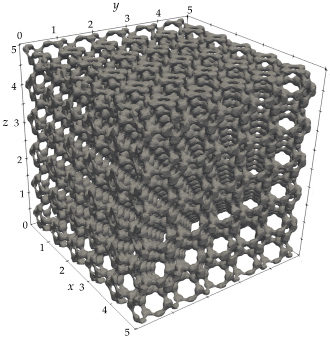


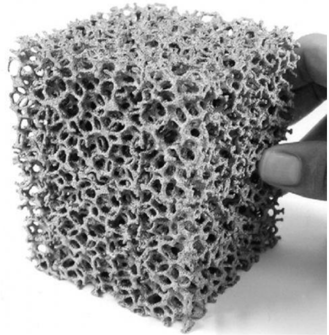
D Le Hardy, M.A. Badri, B. Rousseau, S. Chupin, D. Rochais et Y. Favennec. "3D numerical modelling of the propagation of radiative intensity through a X-ray tomo- graphied ligament". In : Journal of Quantitative Spectroscopy and Radiative Transfer 194 (2017), p. 86–97

Structured Kelvin cell foam

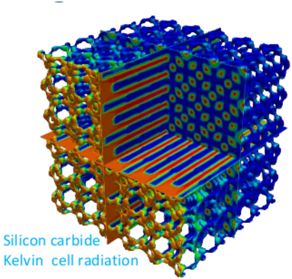
- Virtually created 5x5x5 Kelvin cell (genMat) studied
- Preprocessed with **surface mesh adaption**
- Mesh **15 million nodes** & DOM $N_d = 512$ $\Longrightarrow$ **8 billion unknowns**

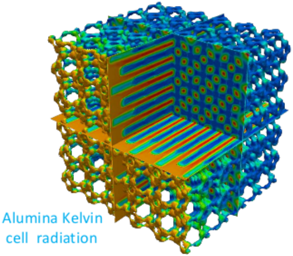
- **1200 MPI** processes on ICI supercomputer LIGER

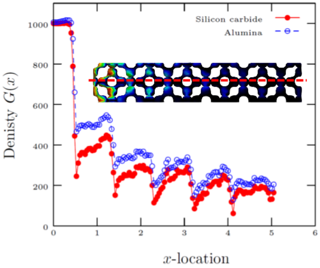




application (512 equations ; 8 billion unknowns)

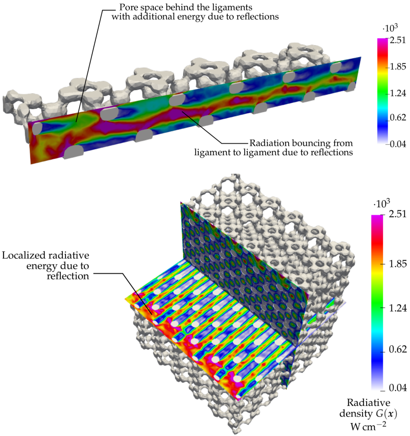


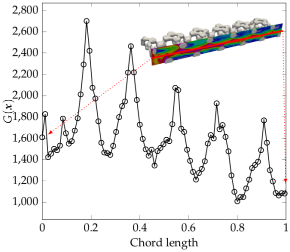




- Step wise decrease in energy
- Scattering alteration
- **Solved in less than 15 min**

Structured Kelvin cell foam



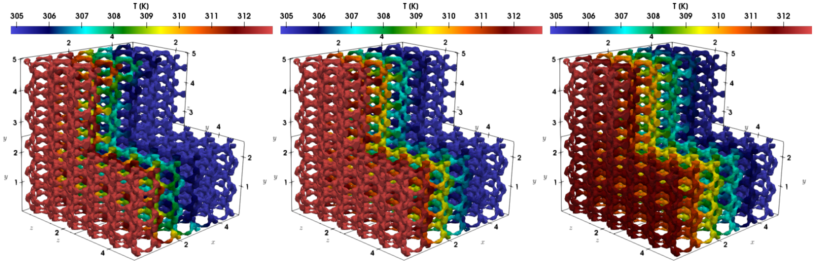


- SiC foam reflection creates a global backscatter

conclusion

- FEM pour l'ETR
  - DOM – vectorial FEM
  - stabilisation SUPG
  - Gestion de la spécularité
  - décomposition de domaine
  - GMRES – BiCGStab, préconditionnement
  - applications à des géométries complexes
- méso-centres de calcul :
  - LIGER (Centrale Nantes)
  - CCIPL (Univ. Nantes)
- avec, bien sûr :
  - M.A. Badri (doctorant IRT Jules-Verne, 2015–2018)
  - D. Le Hardy (doctorant MESR LTeN, 2013–2016)
  - P. Jolivet (Institut Recherche Informatique Toulouse, ENSEEIHT)
- Next : linéaire / non linéaire / couplages / physique / dépôt du solveur

conducto-radiatif



$t = 0.0$ s $t = 1.0$ s $t = 2.0$ s