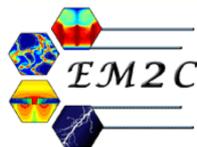


A three-equation model for the prediction of soot emissions in LES of gas turbines

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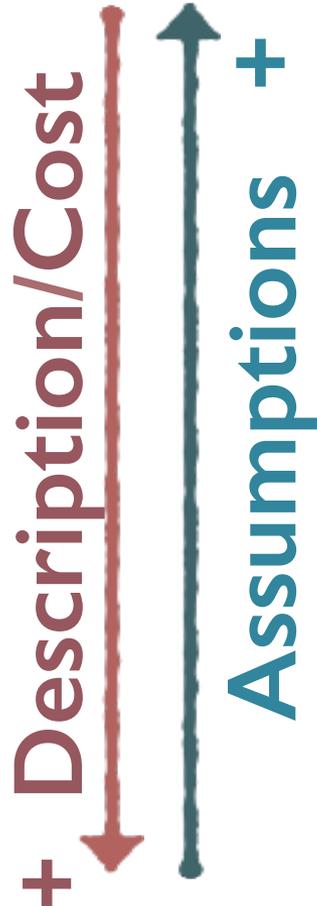


Context – Eulerian soot modeling

Design of low-emission burners = LES param. studies of real configurations

→ Development of soot models:

- Accounting for poly-disperse population of particles
- Minimal CPU cost



Semi-empirical models (2eqns)

- + *Access to global quantities*
- *No access to PSD functions*
- *Empirical relations for soot source terms*

Method of moments (~5eqns)

- + *Detailed description of soot production process*
- *Presumed PSDF shape*
- *Closure models and numerical issues*

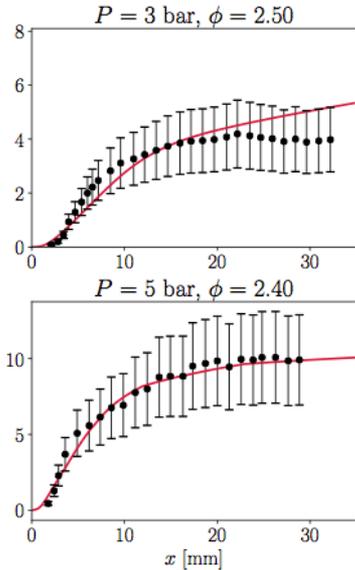
Sectional models (~25eqns)

- + *Detailed description of soot production process*
- + *Low assumptions on PSDF shape*
- + *Converges towards a continuous description*
- *CPU cost*

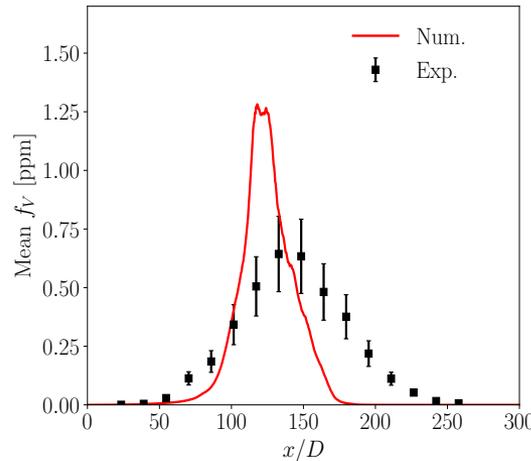
M.E. Mueller and H. Pitsch, *Physics Fluids* (2013); G. Lecocq et al., *Flow, Turb., Combust.* (2014); M. Grader et al., *ASME Turbo Expo* (2018); H. Koo et al., *AIAA* (2015); P. Rodrigues et al. *Comb. Flame* (2018).

State-of-art on LES of sooting turbulent flames

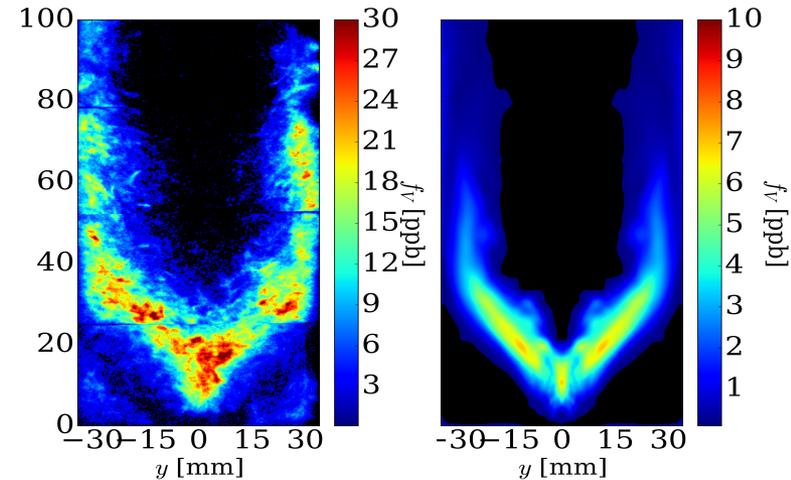
A sectional method for soot has been developed and validated on numerous flames^[1]



Laminar premixed flames
($P=1,3,5$ bar)



Turbulent jet flame



Turbulent swirled flame

Gas phase: 10eqns (NS + look-up table)

Solid phase: 30eqns (PAHs+subgrid+sections)

➔ High CPU cost: no parametric/industrial studies

OBJECTIVE

Development of a reliable but cheap model:

- Reduced model (*simpler formulation, physical understanding and implementation*)
- Post-processing PSD reconstruction

[1] P. Rodrigues et al., *Proc. Comb. Inst.* (2017); P. Rodrigues et al., *Comb. Flame* (2018).

Three-equation model

NDF *Bi-variate number density function*

Global quantities

$$f(t, x, v, s) \left\{ \begin{array}{l} N_s = \langle f \rangle \quad \text{Total number density} \\ f_v = \langle v f \rangle \quad \text{Soot volume fraction} \\ S_s = \langle s f \rangle \quad \text{Total soot surface} \end{array} \right.$$
$$\langle \Phi \rangle = \int \int \Phi dv ds$$

→ *Soot mass fraction* $Y_s = \rho_s \rho^{-1} f_v$

Mean particle volume and surface $v_s = f_v N_s^{-1}$ $s_s = S_s N_s^{-1}$

Transport equation

$$\frac{\partial \Psi}{\partial t} + \nabla \cdot (\mathbf{u} \Psi) = \nabla \cdot \left(C_{th} \nu \frac{\nabla T}{T} \Psi \right) + \dot{\omega}_{\Psi}$$

The source terms are not fitted but derived from the sectional formulation → **NOT** a semi-empirical model!

Closure of source terms

The source terms are not fitted but derived from the sectional formulation by **assuming a mono-disperse distribution**

$$f(v, s) = N_s \delta_{v_s} \delta_{s_s}$$

CONDENSATION

$$\frac{\dot{\omega}_{Y_s}}{\rho_s} = \boxed{v_{\text{dim}} \beta_{v_{\text{dim}}}^{\text{fm}} N_{\text{dim}}^2} + \boxed{v_{\text{dim}} \beta_{v_{\text{dim}}, v_s}^{\text{fm}} N_{\text{dim}} N_s} + v_{C_2} \lambda (k_{\text{sg}} - k_{\text{ox}}) s_s N_s$$

COAGULATION

$$\dot{\omega}_{N_s} = \boxed{\frac{\beta_{v_{\text{dim}}}^{\text{fm}} N_{\text{dim}}^2}{2}} \text{ **NUCLEATION** } - (1 - \mathcal{H}[v_s - v_{C_2}]) \lambda k_{\text{ox}} s_s N_s \boxed{\frac{\beta_{v_s} N_s^2}{2}}$$

$$\dot{\omega}_{S_s} = \boxed{(18\pi)^{1/3} (v_{\text{dim}})^{2/3} \beta_{v_{\text{dim}}}^{\text{fm}} N_{\text{dim}}^2} + \boxed{\delta s_{v_{\text{dim}}}^{\text{frac}} \beta_{v_{\text{dim}}, v_s}^{\text{fm}} N_{\text{dim}} N_s} + \lambda (\delta s_{v_{C_2}}^{\text{frac}} k_{\text{sg}} - \delta s_{v_{C_2}}^{\text{spher}} k_{\text{ox}}) s_s N_s$$

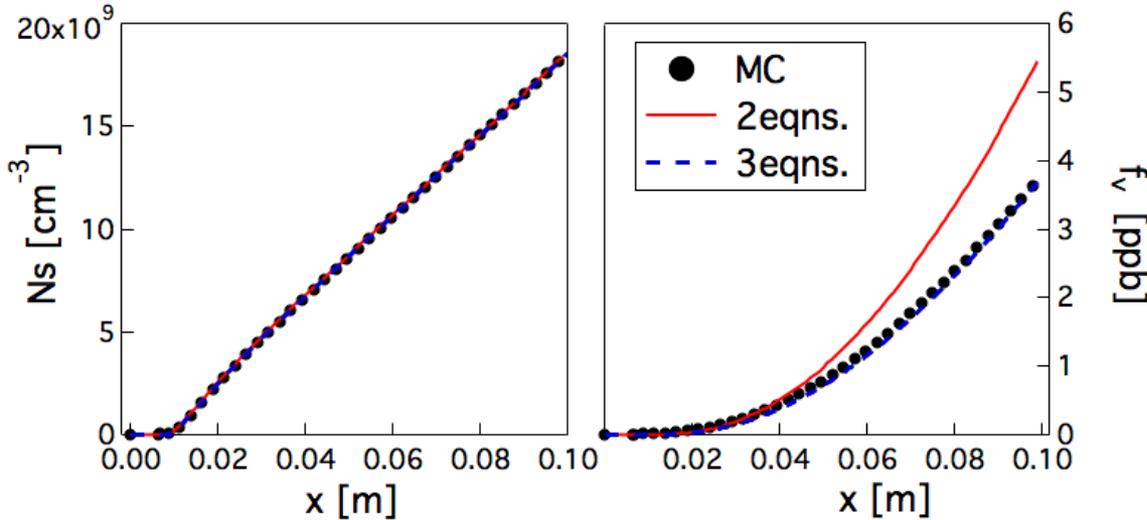
Validation for global quantities: premixed flame

Comparing with reference description (Monte-Carlo)



Monte-Carlo Sweep code^[1]

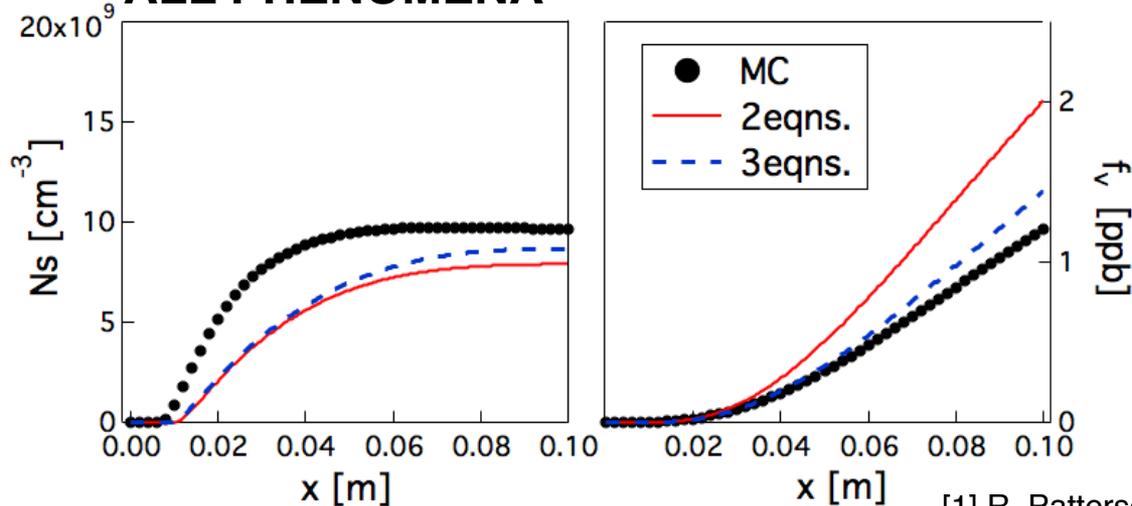
NUCLEATION & SURF. GROWTH



$$S_s^{2eq} = (36\pi)^{1/3} v_s^{2/3}$$
$$S_s^{3eq} = S_s N_s^{-1}$$

Equation for surface improves the accuracy

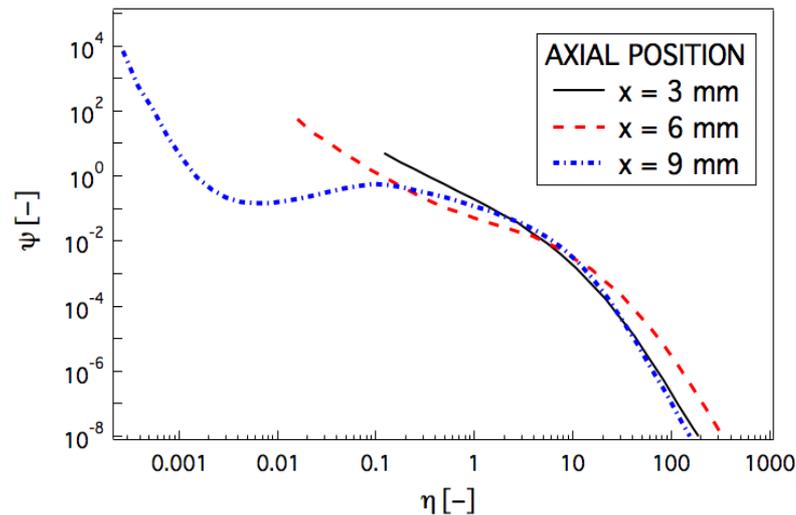
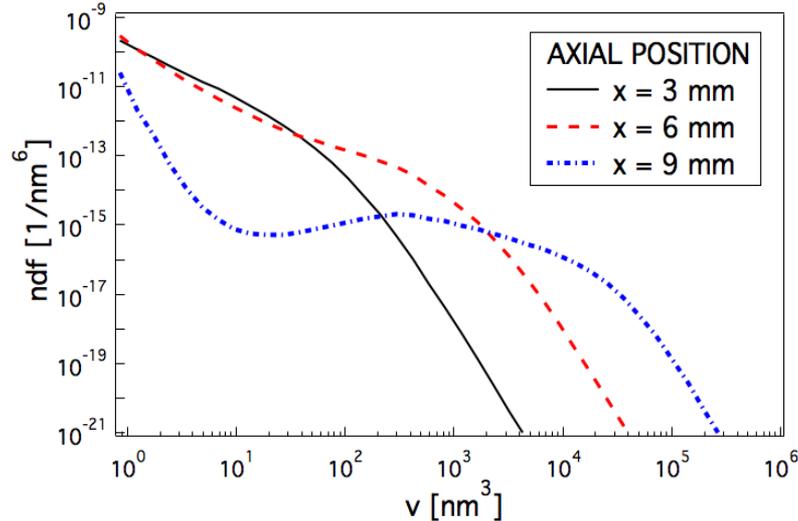
ALL PHENOMENA



- Reasonable description with 3-eqns
- Discrepancies are found for bimodal distributions

[1] R. Patterson et al. *J. SIAM on Scientific Computing* (2006).

Marginal NDF



$$n(v) = \int f(v, s) ds$$

Results for a laminar premixed flame^[1] using a sectional method^[2]

DIMENSIONLESS NDF

$$\psi(\eta) = n f_v / N_s^2$$

$$\eta = v / v_s$$

Self-similarity^[3] does not apply to soot NDF

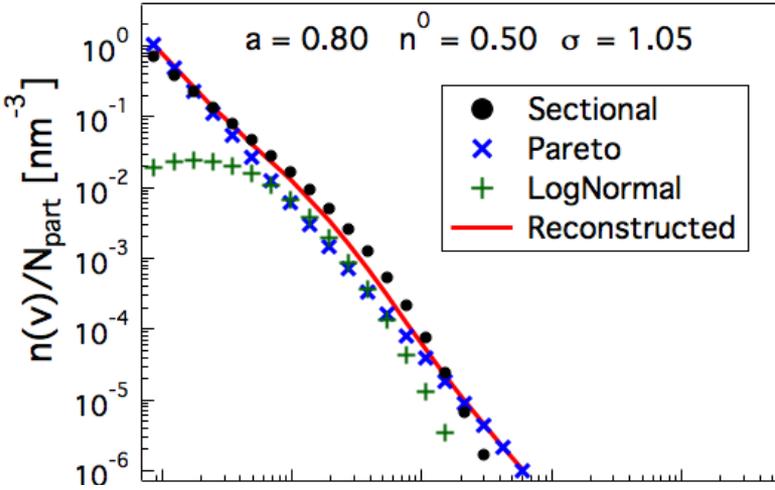
[1] A.D. Abid et al., *Comb. Flame* (2008).

[2] P. Rodrigues et al. *Proc. Comb. Inst.* (2017).

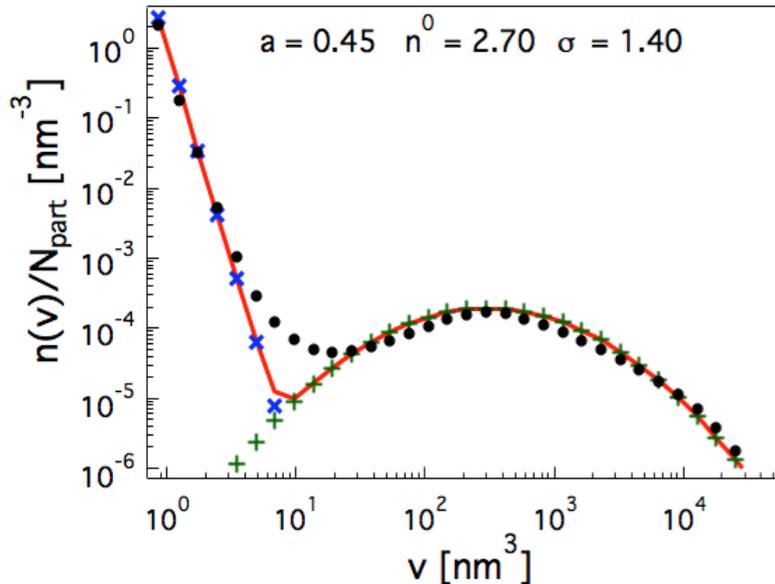
[3] F. Jun et al. *Fire Sci.* (2004).

Reconstruction of the NDF

$x = 3 \text{ mm}$



$x = 9 \text{ mm}$



Reconstructed NDF (R-NDF)

$$\bar{n}(v) = \frac{n(v)}{N_s} \approx a \bar{n}_1(v) + (1 - a) \bar{n}_2(v)$$

Pareto
LogNormal

Distributions parameters are analytically derived except for:

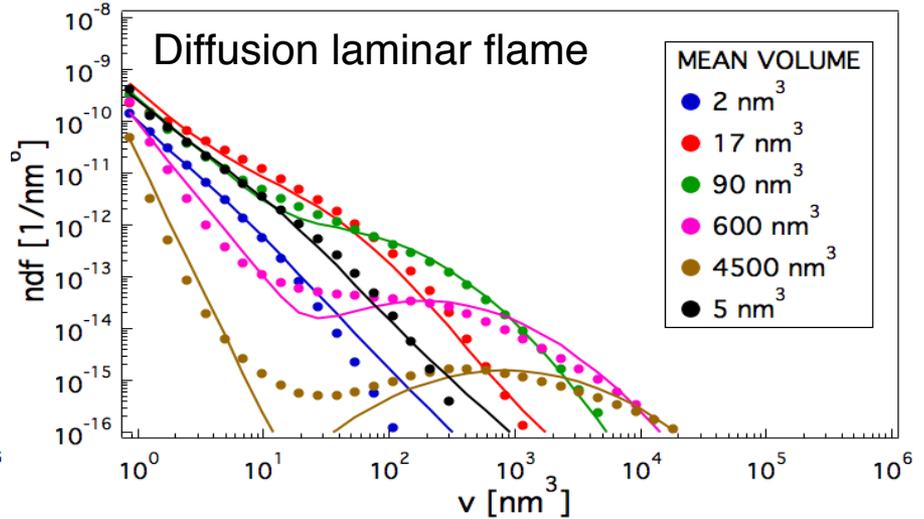
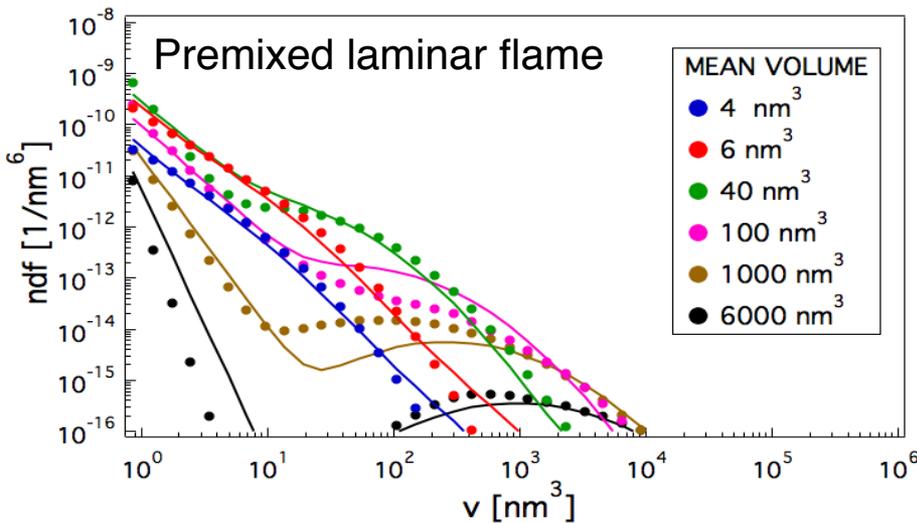
$$a = \max \left[0, 1.0 - 0.18 \left(\frac{v_s}{v_{\text{nucl}}} \right)^{0.12} \right]$$

$$\bar{n}^0 = 8(1 - a)^2 \quad \sigma = 1 + 0.65(1 - a)$$

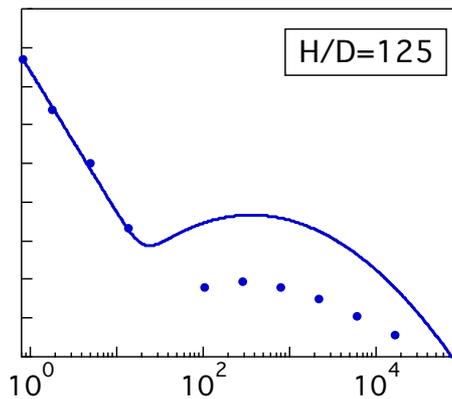
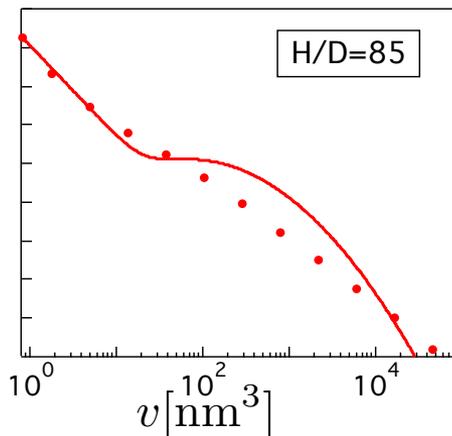
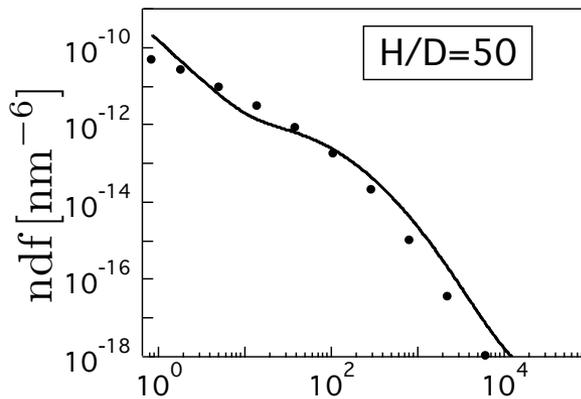
v_s determines the R-NDF shape:

- Small v_s (f_v): one-peak
→ Pareto
- Large v_s (f_v): two-peaks
→ Pareto+Lognormal

A priori R-NDF validation: sec (sym) vs 3eqns (line)



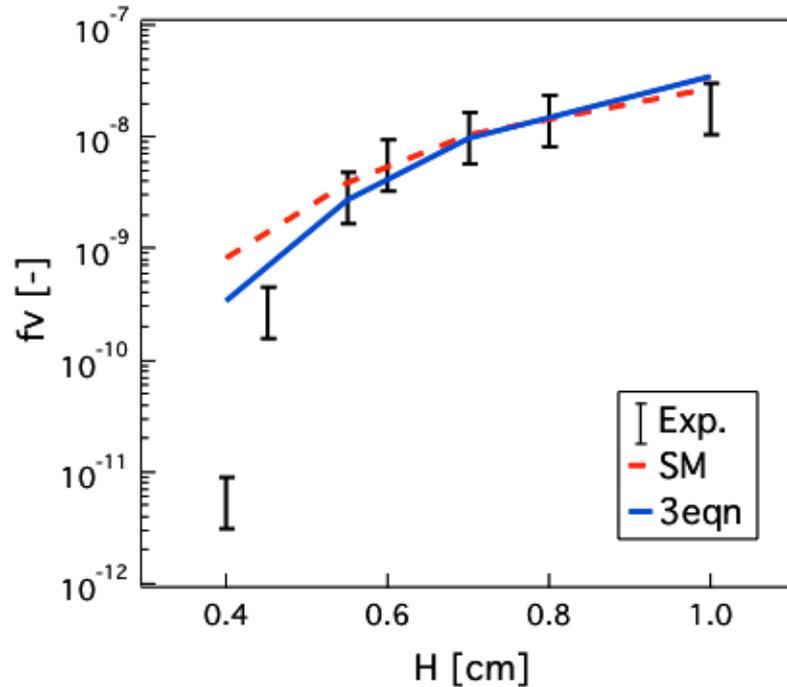
Turbulent jet flame (instantaneous results)



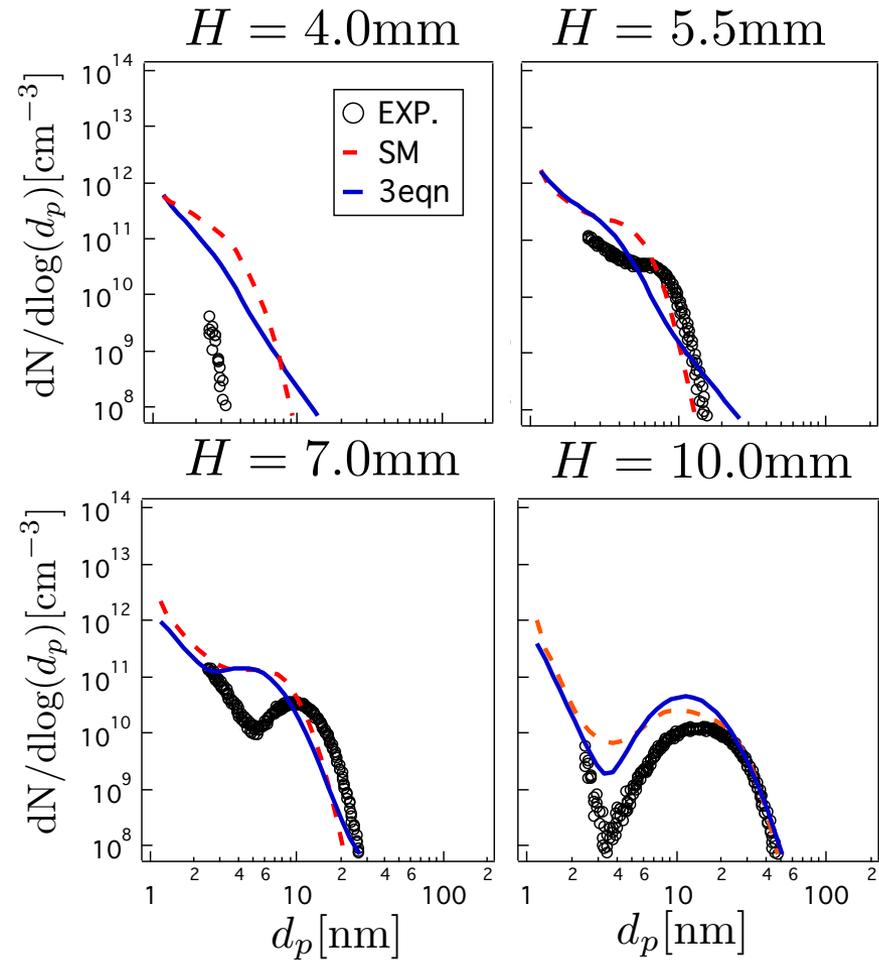
- R-NDF correctly reproduces the ndf-shape (one-/two-peaks).
- Empirical R-NDF relations → dependence on fuel/operating conditions BUT decomposition into Pareto and logNormal seems general.

A posteriori validation: laminar-ISF benchmark

Burner Stabilized Stagnation laminar premixed oxygen/argon/ethylene flame^[1,2]



- Good agreement with SM and experiments.
- Accuracy is representative of the state-of-art soot modeling.



[1] C. Saggese et al. *Comb. Flame* (2015).

[2] P. Rodrigues et al. *Proc. Comb. Inst.* (2017).

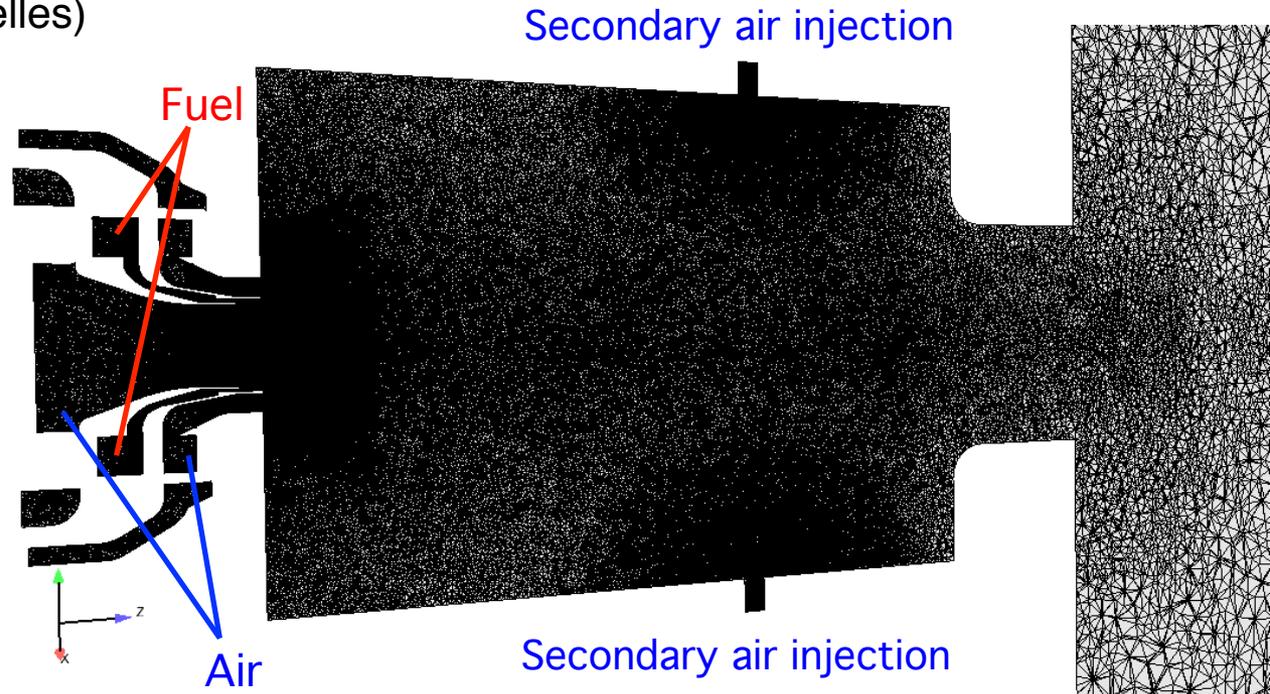
A posteriori validation: DLR^[1]-ISF benchmark

Solver: AVBP

(CERFACS/IFP Energies nouvelles)

Setup already validated for SM

- *Gas phase: RFPV model from KM2 mechanism + beta-pdf*
- *Solid phase: intermittency model*
- *Radiation: optically-thin model*
- *Experimental temperature imposed on chamber wall*



Operating point

T=300 K

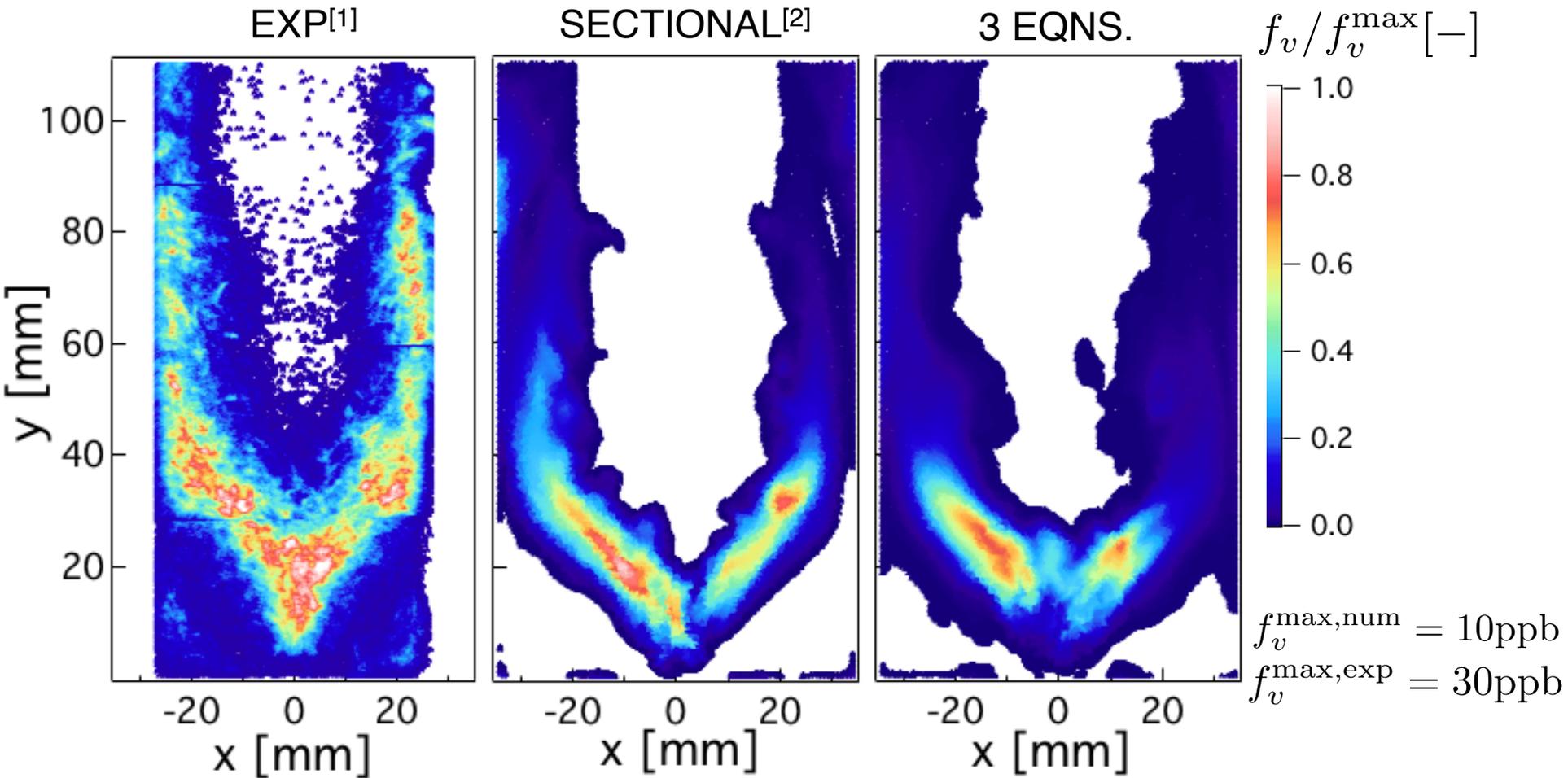
P=3 bar

$Q_{\text{oxi}}/Q_{\text{air}}=0.4$

Scheme	TTGC: 3 nd order in time and space
Cells	40,000,000
SGS model	WVALE
Av. time	30ms

[1] K.P. Geigle et al. *ASME Turbo Expo* (2011).

LES results on FIRST test rig

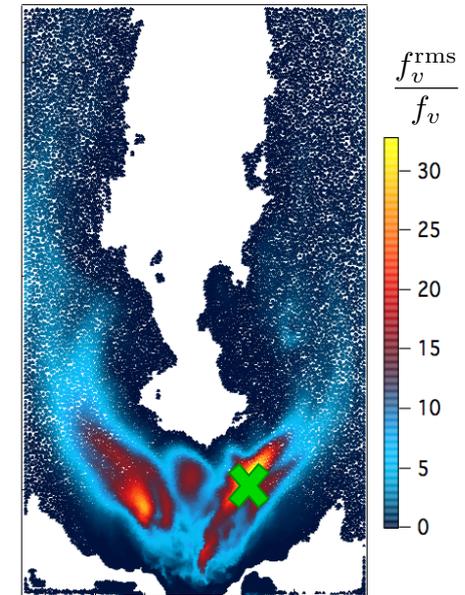
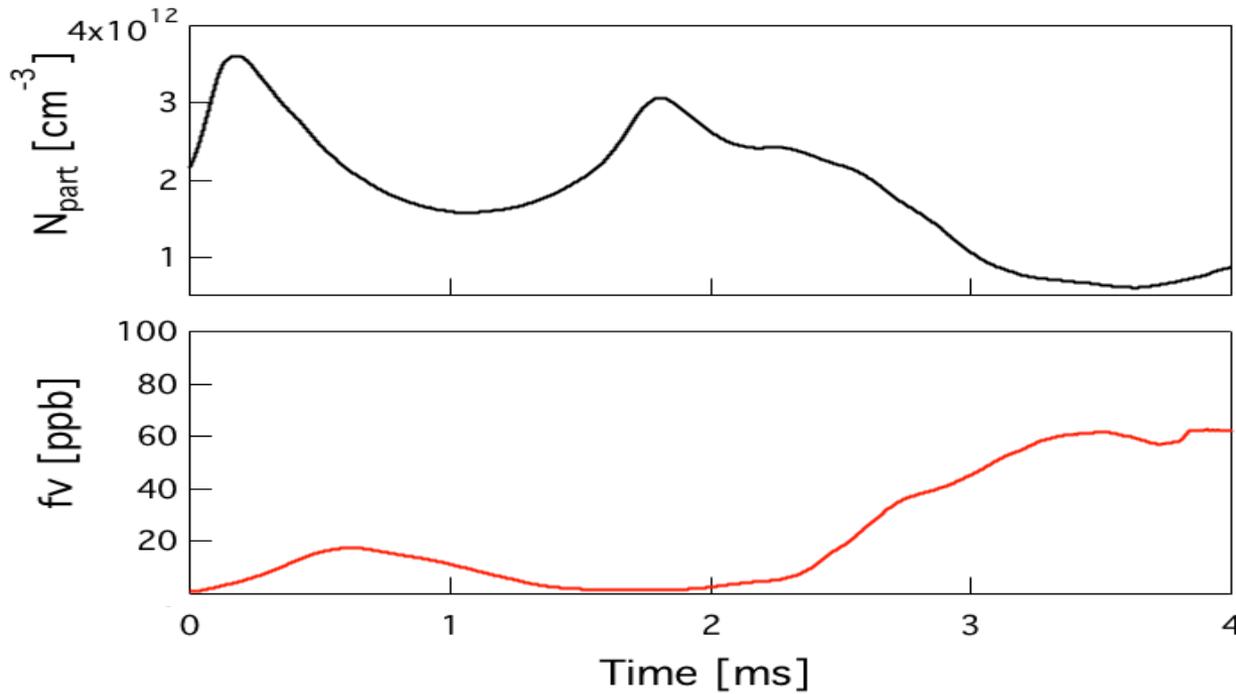


- Good agreement with experiments (soot yield and position)
- Satisfactory agreement sectional VS 3eqns (and CPU cost reduced by 3!)

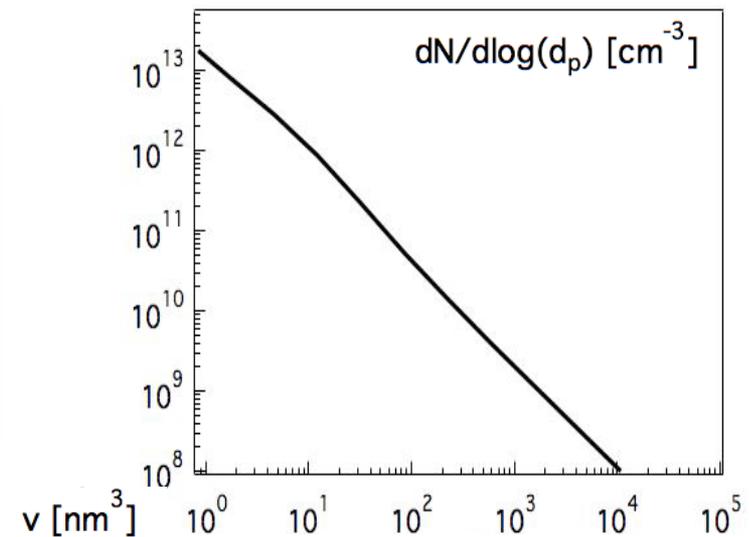
[1] K.P. Geigle et al. *ASME Turbo Expo* (2011).

[2] P. Rodrigues et al. *INCA* (2017).

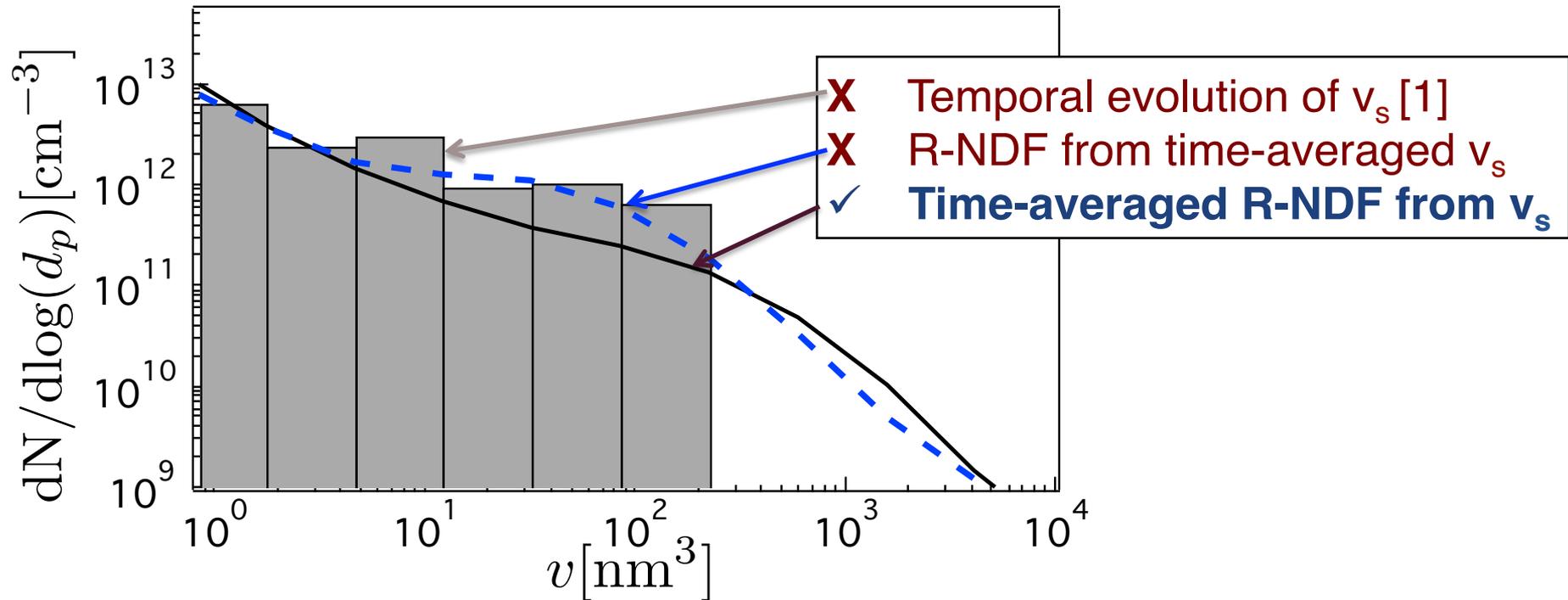
High fluctuations of soot quantities



- PSD strongly varies in space and time
- Time-averaged PSD is almost everywhere one-peak (as for sectional)



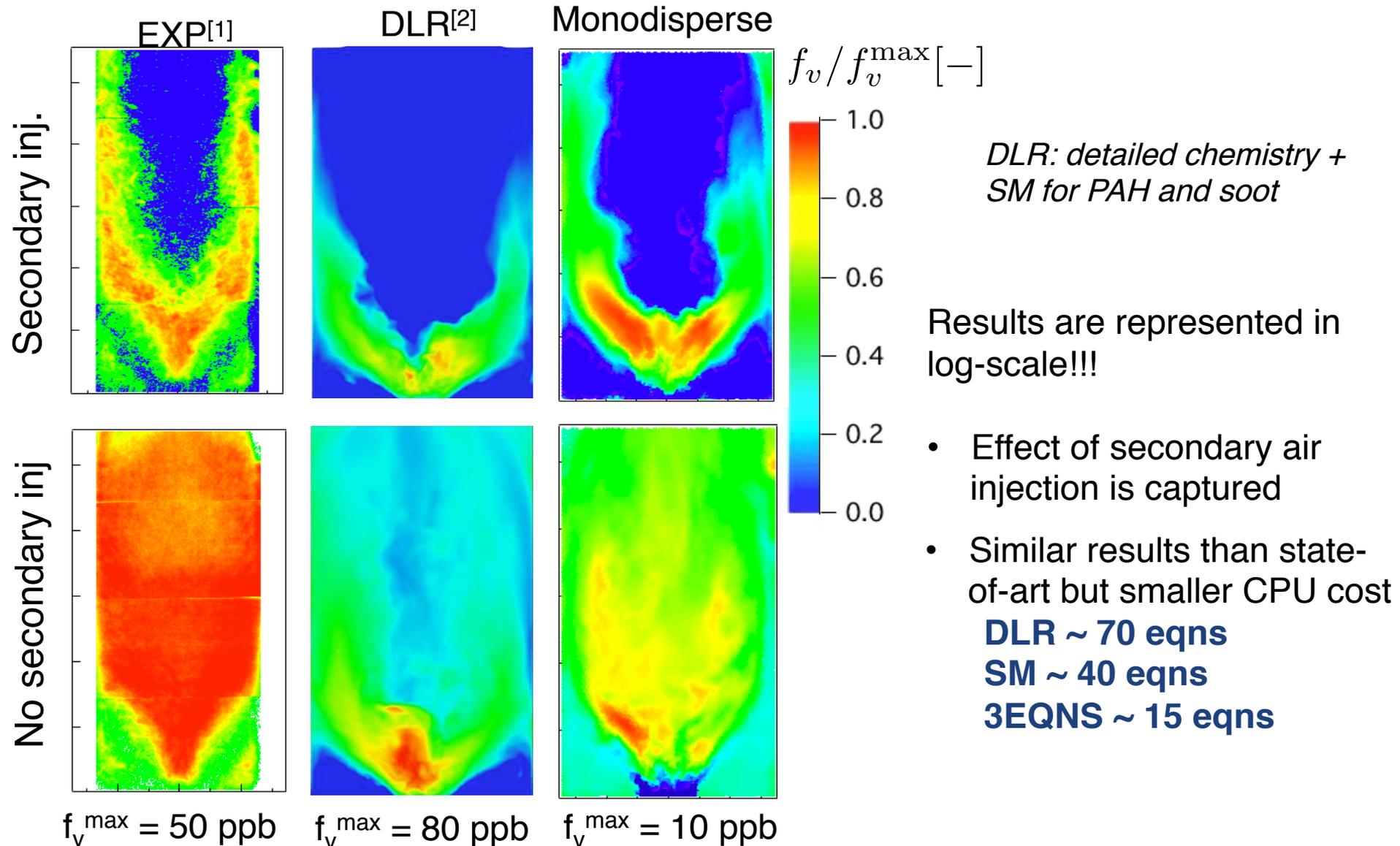
Have to obtain time-averaged PSD?



High non-linearity of PSD with respect to moments:
the temporal evolution of the PSD (reconstructed or
transported) has to be averaged during the computation

*N.B. No PSD validation is provided by comparison with
experimental data (not available for this configuration)*

Secondary air injection effect



[1] K.P. Geigle et al. *ASME Turbo Expo* (2013)

[2] M. Grader et al., *ASME Turbo Expo* (2018) **14**

Conclusion



Development of a reliable soot model for LES of gas turbine:

1. *New equation for total soot surface: better description of surface reactions*
2. *Theoretical development: larger validity compared to empirical models*
3. *Easy implementation into CFD solver*
4. *Reasonable estimation of the PSD*
5. *Low CPU cost*

- R-NDF requires more validation
- 3eqns accuracy is reduced when NDF is highly poly-disperse.

STILL



Good candidate for LES of gas turbines: fv small ($< 1\text{ppm}$)
→ monodisperse population.

Acknowledgement

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